# CATEGORY THEORY
# CATEGORY II - EQUISETS

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## 1. Equisets

**Definition 1** (Objects)**.** An *equiset* $(E, \sim)$ consists of a set $E$ together with a relation $\sim$ on $E$ with the following properties:

| | | |
|---|---|---|
| **(E1)** | $a \sim a$ | (Reflexivity) |
| **(E2)** | $a \sim b$ implies $b \sim a$ | (Symmetry) |
| **(E3)** | $a \leq b$ and $b \leq c$ implies $a \leq c$ | (Transitivity) |

The relation $\sim$ is called an *equivalence relation* on $E$.

**Definition 2** (Subobjects)**.** Let $(E, \sim)$ be an equiset. If $F \subset E$, the restriction of $\sim$ to $F$ satisfies the properties of an equivalence relation on $F$, making $(F, \sim)$ an equiset. We may call $(F, \sim)$, or just $F$, a *subequiset*.

**Definition 3** (Morphisms)**.** Let $(E, \sim)$ and $(F, \approx)$ be equisets. A function $f : E \to F$ is called *equivalence preserving* if

$$e_1 \sim e_2 \quad \Rightarrow \quad f(e_1) \approx f(e_2).$$

The identity map on $P$ is order preserving, and the composition of order preserving functions is order preserving. Thus, posets with order preserving maps form a category.

**Problem 1.** Discuss when a function $f : (\mathbb{Z}, \equiv_n) \to (\mathbb{Z}, \equiv_m)$ is equivalence preserving, where

$$a \equiv_n b \quad \Leftrightarrow \quad n \mid b - a.$$

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